

DISCRETE MATHEMATICS WITH APPLICATIONS 3RD EDITION File PDF

Discrete Mathematics with Applications 3rd Edition is a comprehensive textbook that serves as an essential resource for students and professionals seeking to understand the fundamental principles of discrete mathematics and its wide-ranging applications. This edition, authored by Kenneth H. Rosen, is renowned for its clear explanations, practical examples, and a balanced focus on theory and real-world applications. Whether you are a computer science student, a software engineer, or someone interested in mathematical logic, this book provides valuable insights into the discrete structures that underpin modern computing and information systems.

Overview of Discrete Mathematics

Discrete mathematics is a branch of mathematics dealing with discrete elements that use algebra and arithmetic. Unlike continuous mathematics, which involves calculus and analysis, discrete mathematics focuses on countable, distinct objects. Its applications are pivotal in computer science, cryptography, network theory, and combinatorics.

Core Topics Covered

Discrete Mathematics with Applications 3rd Edition covers several fundamental topics, including:

- Logic and proof techniques
- Set theory and relations
- Functions and algorithms
- Number theory and cryptography
- Combinatorics and counting principles
- Graph theory and trees
- Algorithms and complexity

Each chapter is designed to build upon previous concepts, ensuring a structured learning experience.

Key Features of the 3rd Edition

The third edition of *Discrete Mathematics with Applications* enhances previous content with updated examples, additional exercises, and clearer explanations. Some notable features include:

- Real-world applications to demonstrate the relevance of concepts
- Chapter summaries and review questions for reinforcement
- Supplementary material such as online resources and instructor's solutions manual
- Focus on problem-solving skills essential for academic and professional success

Applications of Discrete Mathematics

Discrete mathematics is not just an academic subject; it forms the backbone of many technological innovations and practical applications.

Computer Science and Programming

Algorithms, data structures, and programming languages rely heavily on discrete mathematics concepts. For example:

- Graph algorithms for network routing and social network analysis
- Boolean algebra in circuit design and digital logic
- Recursion and functions in programming paradigms

Cryptography and Security

Number theory and modular arithmetic are crucial for encryption algorithms such as RSA and ECC (Elliptic Curve Cryptography). Discrete mathematics provides the tools to develop secure communication protocols.

Network Theory and Data Structures

Graph theory helps model and analyze networks—be it the internet, transportation systems, or social networks. Trees, bipartite graphs, and other structures are fundamental in organizing data efficiently.

Combinatorics and Optimization

Counting principles and combinatorial analysis are used in resource allocation, scheduling, and probabilistic modeling, which are essential in operations research and logistics.

Importance of Learning Discrete Mathematics

Studying discrete mathematics equips learners with critical thinking and problem-solving skills. It fosters logical reasoning, analytical skills, and the ability to model complex systems mathematically.

Why Choose *Discrete Mathematics with Applications 3rd Edition*

This textbook is highly recommended because of its:

- Clear and concise explanations of complex topics
- Rich array of exercises ranging from basic to challenging
- Illustrative examples demonstrating real-world applications
- Focus on developing both theoretical understanding and practical skills

Supplementary Resources and Learning Aids

To maximize learning, students and instructors can utilize:

- Online companion websites with additional exercises and solutions
- Video lectures and tutorials associated with the textbook
- Study guides and practice exams for self-assessment

These resources help clarify difficult concepts and provide opportunities for hands-on practice.

Who Should Use This Book?

Discrete Mathematics with Applications 3rd Edition is suitable for:

- Undergraduate students in computer science, mathematics, or engineering
- Graduate students seeking a solid foundation in discrete math
- Professionals working in software development, cybersecurity, or data analysis
- Educators and instructors designing course curricula

The book's approachable style makes complex topics accessible to a broad audience.

Conclusion

In summary, **discrete mathematics with applications 3rd edition** is an essential resource that bridges theoretical concepts with practical applications across various fields. Its comprehensive coverage, coupled with clear explanations and real-world relevance, makes it an invaluable tool for anyone interested in understanding the mathematical structures that underpin modern technology. Whether you are a student aiming to excel in your coursework or a professional seeking to deepen your knowledge, this edition provides the necessary foundations to succeed and innovate in the

digital age.

Discrete Mathematics with Applications 3rd Edition: An In-Depth Review

Discrete mathematics forms the backbone of computer science, cryptography, combinatorics, and many other technological fields. Among the myriad textbooks available, Discrete Mathematics with Applications, 3rd Edition stands out as a comprehensive resource, blending rigorous theory with practical application. This review aims to explore the core features, pedagogical strengths, and real-world relevance of this book, helping educators, students, and professionals understand its value and utility.

Overview and Scope of the Book

Discrete Mathematics with Applications, 3rd Edition is authored by Susanna S. Epp, a renowned figure in mathematics education. The book is designed as an accessible yet thorough introduction to discrete mathematics, emphasizing both conceptual understanding and practical problem-solving skills.

Key Features:

- Covers fundamental topics such as propositional logic, predicate logic, set theory, functions, relations, and algorithms.
- Includes chapters on combinatorics, number theory, graph theory, and discrete probability.
- Features numerous real-world applications, illustrating how discrete math principles underpin computer science, information theory, and cryptography.
- Incorporates a variety of exercises, from straightforward practice problems to challenging proofs, fostering deep comprehension.

The third edition has been refined to align with current pedagogical standards, with updated examples, clearer explanations, and expanded exercises.

Pedagogical Approach and Structure

Clear Explanations and Logical Progression

One of Epp's distinguishing features is her clear, student-friendly writing style. The textbook is structured to facilitate gradual learning, beginning with foundational concepts and building toward more complex topics.

- Chapter Organization: Each chapter introduces key concepts with definitions, followed by illustrative examples. Theoretical results are accompanied by proofs that are carefully explained to avoid assumptions of prior advanced knowledge.

- Incremental Complexity: Concepts such as propositional logic are introduced with simple truth tables, then extended into more sophisticated topics like logical inference and proof techniques.

- Practical Applications: Interspersed throughout are real-world scenarios, such as network security, data encoding, or algorithm efficiency, showing students how abstract ideas translate to tangible outcomes.

Variety of Exercises and Reinforcement

The book employs an array of exercises to reinforce learning:

- Routine Problems: Basic questions designed to test understanding of definitions and simple applications.

- Challenging Proofs: Longer exercises that require constructing formal proofs, encouraging mastery of logical reasoning.

- Applied Problems: Situations based on real-world contexts, fostering critical thinking about the relevance of discrete math.

- Group Activities: Some sections suggest collaborative problem-solving, promoting collaborative learning.

This diversity ensures students develop both computational skills and theoretical insights.

Content Deep Dive and Key Topics

Propositional and Predicate Logic

The book begins with the essentials of propositional logic, covering:

- Logical connectives (and, or, not, implies, if and only if)
- Truth tables and logical equivalences
- Normal forms (conjunctive and disjunctive)
- Logical inference rules and proofs

Predicate logic extends these ideas with quantifiers, predicates, and the notion of logical validity, forming the foundation for formal reasoning and computer program verification.

Set Theory and Combinatorics

Set theory is presented as a unifying language:

- Basic set operations (union, intersection, difference)
- Cartesian products and power sets
- Venn diagrams and their applications

In combinatorics, topics include:

- Permutations and combinations
- Binomial theorem
- Pigeonhole principle
- Inclusion-exclusion principle

These topics are crucial in probability calculations, algorithm analysis, and resource allocation problems.

Functions, Relations, and Algorithms

Understanding functions and relations is vital:

- Types of functions (injective, surjective, bijective)
- Equivalence relations and partitions
- Partial and total orders

Algorithms are introduced through recursive definitions, asymptotic notation, and complexity analysis, connecting discrete math to computer science.

Number Theory and Cryptography

Number theory topics include:

- Divisibility, primes, Euclidean algorithm
- Modular arithmetic and Fermat's little theorem

The book explores cryptography applications, illustrating how discrete math principles underpin secure communication protocols.

Graph Theory and Discrete Probability

Graph theory covers:

- Graph representations, paths, cycles
- Connectivity, trees, and spanning trees
- Planar graphs and graph coloring

Discrete probability addresses:

- Basic probability rules
- Conditional probability
- Expected value and variance

These areas are fundamental for network design, data analysis, and randomized algorithms.

Applications and Real-World Relevance

Discrete Mathematics with Applications excels at bridging theory with practice, demonstrating how discrete math concepts are integral in numerous fields:

- Computer Science: Algorithm design, complexity theory, data structures, and programming language semantics.
- Cryptography: Encryption algorithms, key exchange protocols, digital signatures.
- Network Design: Graph algorithms for routing, network reliability, and social network analysis.
- Data Science: Probabilistic models, combinatorial optimization, and data encoding.
- Logic and Formal Methods: Software verification, automated reasoning, and artificial intelligence.

By showcasing these applications, the book motivates learners and provides context that enhances understanding.

Strengths and Pedagogical Effectiveness

Strengths:

- Clarity and Accessibility: Epp's writing demystifies complex topics, making the material

approachable for undergraduates and newcomers.

- Comprehensive Coverage: The book balances breadth and depth, ensuring a solid foundation in core topics while also exploring advanced applications.
- Emphasis on Proofs: Students develop rigorous reasoning skills, essential for theoretical computer science and mathematics.
- Real-World Examples: Connecting abstract concepts to practical scenarios fosters engagement and demonstrates relevance.
- Rich Exercise Set: A wide array of problems supports diverse learning styles and mastery levels.

Potential Limitations:

- Some learners may find the depth of proofs challenging initially.
- The density of material requires dedicated effort; pacing may vary depending on prior background.

Conclusion: Who Should Use This Book?

Discrete Mathematics with Applications, 3rd Edition is an outstanding resource for:

- Undergraduate students in computer science, mathematics, or engineering seeking a thorough introduction.
- Instructors aiming for a textbook that balances theory with applications and offers ample exercises.
- Self-learners interested in gaining foundational knowledge with practical relevance.

Its combination of clear explanations, real-world applications, and rigorous exercises makes it a valuable asset in any discrete mathematics curriculum.

Final Thoughts

In an era where digital systems increasingly underpin our daily lives, understanding the principles of discrete mathematics is more important than ever. Epp's Discrete Mathematics with Applications, 3rd Edition delivers this understanding through a carefully crafted blend of theory, application, and pedagogy. Whether used as a textbook or a reference guide, it equips learners with the logical tools necessary to navigate and innovate in the digital age.

Rating: 4.5/5 ? Highly recommended for its clarity, depth, and practical relevance.

QuestionAnswer What are the key topics covered in 'Discrete Mathematics with Applications, 3rd Edition'? The book covers fundamental topics such as logic, set theory, combinatorics, graph theory, algorithms, and discrete probability, along with their practical applications. How does the 3rd edition of 'Discrete Mathematics with Applications' enhance understanding for students? It includes updated examples, new exercises, real-world application problems, and clearer explanations to help students grasp complex concepts more effectively. Are there digital resources or supplementary materials available for this edition? Yes, the 3rd edition offers online resources such as solution manuals, lecture slides, and interactive exercises to support learning and teaching. What are some practical applications of discrete mathematics highlighted in this textbook? The textbook emphasizes applications in computer science, cryptography, network design, data analysis, and algorithm development. Is 'Discrete Mathematics with Applications, 3rd Edition' suitable for self-study? Yes, its clear explanations, numerous exercises, and real-world examples make it a suitable resource for self-learners interested in discrete mathematics. How does this edition compare to previous editions in terms of content updates? The 3rd edition features updated chapters with current topics like advanced graph algorithms, expanded problem sets, and recent applications to reflect the latest developments in the field.

Related keywords: discrete mathematics, combinatorics, graph theory, logic, set theory, algorithms, mathematical proofs, discrete structures, number theory, applications in computer science

discrete probability models and methods probabili

discrete probability models and methods probabili form a foundational pillar in the field of probability theory and statistics. These models are essential for understanding and analyzing situations where the set of possible outcomes is countable or finite, such as rolling dice, flipping coins, or drawing cards from a deck. By employing discrete probability models, statisticians and mathematicians can quantify uncertainty, make predictions, and derive meaningful inferences from data. This article delves into the core concepts, common models, methods for analysis, and practical applications of discrete probability models and methods probabili.

Understanding Discrete Probability Models

Discrete probability models are mathematical frameworks used to describe random experiments with discrete outcomes. These models assign probabilities to each possible outcome, adhering to specific axioms that ensure the probabilities make sense within the context of real-world scenarios.

Basic Concepts and Definitions

- **Sample Space (Ω):** The set of all possible outcomes of a random experiment. For discrete models, Ω is countable (finite or countably infinite).
- **Event:** Any subset of the sample space. Events can be simple (single outcome) or compound (multiple outcomes).
- **Probability Measure (P):** A function that assigns a probability to each event, satisfying the axioms:
 - Non-negativity: $P(E) \geq 0$ for any event E .
 - Normalization: $P(\Omega) = 1$.
 - Additivity: For mutually exclusive events $E_1, E_2, \dots$, $P(E_1 \cup E_2 \cup \dots) = P(E_1) + P(E_2) + \dots$

Probability Distributions in Discrete Models

Discrete probability distributions specify how the total probability is distributed among the outcomes. Some of the most common discrete distributions include:

- **Uniform Distribution:** All outcomes are equally likely. For a finite set of n outcomes, $P(X = x) = 1/n$.
- **Bernoulli Distribution:** Describes a single trial with two outcomes: success (with probability p) and failure (with probability $1-p$).
- **Binomial Distribution:** Models the number of successes in n independent Bernoulli trials.
- **Poisson Distribution:** Describes the number of events occurring in a fixed interval of time or space when events occur independently.

Common Discrete Probability Models

Each model has specific contexts where it best applies and unique properties that facilitate analysis and computation.

Uniform Distribution

The simplest form, where each outcome has equal probability. Used when outcomes are equally likely, such as rolling a fair die or selecting a random card from a deck.

Bernoulli Distribution

Representing the simplest case of a binary experiment, the Bernoulli distribution models outcomes like coin flips or yes/no responses.

- Probability mass function (PMF): $P(X = x) = p^x (1-p)^{(1-x)}$, where $x \in \{0, 1\}$.

Binomial Distribution

Extends Bernoulli trials to n independent experiments, each with success probability p . It models the total number of successes.

- PMF: $P(X = k) = C(n, k) p^k (1-p)^{(n-k)}$, for $k = 0, 1, \dots, n$.

Poisson Distribution

Ideal for modeling rare events over a continuous domain, such as the number of emails received in an hour or decay events in radioactive substances.

- PMF: $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$, for $k = 0, 1, 2, \dots$

Methods for Analyzing Discrete Probability Models

Analyzing discrete models involves calculating probabilities, expected values, variances, and other statistical measures. Several methods and tools are commonly used.

Probability Calculations

Calculations often rely on the probability mass function (PMF). For compound events, the sum or convolution of individual probabilities is used.

Expected Value and Variance

These measures provide insights into the average outcome and variability:

- **Expected Value (Mean):** $E[X] = \sum x P(X = x)$.
- **Variance:** $\text{Var}(X) = E[(X - E[X])^2] = \sum (x - E[X])^2 P(X = x)$.

Conditional Probability

Understanding the probability of an event given another event is crucial, especially in complex models.

- $P(A | B) = P(A \cap B) / P(B)$, provided $P(B) > 0$.

Independence

Two events A and B are independent if $P(A \cap B) = P(A) P(B)$. Independence simplifies calculations and is a key assumption in many models.

Using Generating Functions

Probability generating functions (PGFs) and moment generating functions (MGFs) are powerful tools for deriving moments and distributions of sums of discrete random variables.

Practical Applications of Discrete Probability Models

These models are instrumental across various industries and fields, enabling better decision-making and risk assessment.

Gaming and Gambling

- Understanding the probabilities in card games, dice, roulette, and lotteries.
- Calculating odds and expected winnings.

Quality Control and Manufacturing

- Modeling defect rates in production lines.
- Determining the likelihood of a certain number of defective items.

Communication Systems

- Analyzing packet loss, error rates, and the number of failures in network systems.

Biology and Epidemiology

- Modeling the number of mutations, disease outbreaks, or survival counts.

Business and Economics

- Forecasting demand, customer arrivals, and inventory levels.

Advanced Topics in Discrete Probability Methods

Beyond basic models, advanced methods enhance analysis and modeling capabilities.

Markov Chains

- Modeling systems that transition between states with certain probabilities.
- Applications include weather prediction, stock market analysis, and queuing systems.

Poisson Processes

- Modeling the occurrence of events over continuous time or space.
- Useful in telecommunications, traffic flow, and reliability engineering.

Multivariate Discrete Distributions

- Handling multiple correlated discrete variables.
- Examples include multinomial distributions and joint probability tables.

Conclusion

Discrete probability models and methods probabili are vital tools for quantifying uncertainty in situations with countable outcomes. They provide a structured approach to calculating probabilities, understanding distributions, and deriving statistical measures essential for decision-making across diverse fields. Mastery of these models enables analysts and researchers to interpret data accurately, develop predictive models, and implement effective strategies in contexts ranging from gaming and manufacturing to healthcare and finance. As the landscape of data analysis continues to evolve, discrete probability models remain a cornerstone of statistical reasoning and probabilistic modeling.

Discrete Probability Models and Methods Probabili: An In-Depth Review

In the realm of probability theory, discrete probability models serve as foundational tools for understanding and analyzing phenomena characterized by countable outcomes. From the simple toss of a coin to complex network models, discrete probability models underpin a vast array of applications across sciences, engineering, economics, and social sciences. This review provides a

comprehensive exploration of discrete probability models and methods probabili, tracing their theoretical underpinnings, key methodologies, practical applications, and recent advancements.

Introduction to Discrete Probability Models

Discrete probability models describe random experiments with a finite or countably infinite set of possible outcomes. Unlike continuous models, which deal with outcomes in an uncountable space, discrete models assign probabilities to individual outcomes, often facilitating exact calculations and interpretations.

Fundamental Concepts

- Sample Space (Ω): The set of all possible outcomes. For discrete models, Ω is countable.
- Event: A subset of the sample space, representing a collection of outcomes.
- Probability Measure (P): A function assigning probabilities to events, satisfying axioms of non-negativity, normalization, and countable additivity.
- Probability Mass Function (PMF): For each outcome ω in Ω , $P(\omega)$ gives its probability; the sum over all outcomes equals 1.

Common Discrete Probability Distributions

- Bernoulli Distribution: Models a single trial with two outcomes, success with probability p , failure with $1-p$.
- Binomial Distribution: Number of successes in n independent Bernoulli trials.
- Geometric Distribution: Number of trials until the first success.
- Negative Binomial Distribution: Number of trials until a fixed number of successes.
- Poisson Distribution: Count of events occurring in a fixed interval, often modeling rare events.

Methodologies and Probabilistic Techniques

Understanding and applying discrete probability models involve various methods that enable analysts to derive probabilities, expectations, variances, and other statistical measures.

1. Enumeration and Combinatorics

Many discrete models rely on combinatorial principles to count the number of favorable outcomes. Techniques include:

- Counting arrangements (permutations)
- Counting subsets (combinations)
- Applying the inclusion-exclusion principle

These methods are essential in deriving explicit formulas for PMFs.

2. Generating Functions

Generating functions encode probability distributions into algebraic forms, facilitating analysis and derivation of moments:

- Probability Generating Function (PGF): $G_X(s) = E[s^X]$
- Use Cases: Deriving moments, convolutions, and limit distributions.

3. Conditional Probability and Bayes' Theorem

Conditional probability is central to models involving dependencies or updates based on new information. Bayesian methods extend discrete models by updating probabilities as evidence accumulates.

4. Markov Chains and Stochastic Processes

Discrete models often extend to sequences of random variables with the Markov property, where the future state depends only on the current state. Applications include:

- Modeling queues
- Population dynamics
- Random walks

Advanced Topics and Methods Probabilistic in Discrete Settings

As the field has evolved, new techniques and models have expanded the scope of discrete probability analysis.

1. Discrete Empirical Distributions

- Used when the underlying distribution is unknown.
- Empirical probabilities are estimated directly from data.

- Essential in non-parametric inference.

2. Approximation Methods

- Poisson Approximation: Approximates binomial distributions when p is small, and n is large.
- Normal Approximation: For large n , binomial and other discrete distributions approximate the normal distribution, via the Central Limit Theorem.

3. Monte Carlo and Simulation Methods

- Use computational algorithms to simulate discrete random variables.
- Useful when analytical solutions are complex or intractable.
- Enable estimation of probabilities, expectations, and variances.

Applications of Discrete Probability Models

Discrete probability models are ubiquitous across disciplines, with applications including:

- Quality Control: Modeling defect counts in manufacturing.
- Epidemiology: Counting disease cases or transmission events.
- Information Theory: Modeling message counts, packet arrivals.
- Economics: Modeling discrete choices, market states.
- Computer Science: Algorithm analysis, randomized algorithms, network modeling.

Recent Advancements and Research Directions

The increasing complexity of real-world data has spurred several recent developments:

1. Discrete Bayesian Models

Bayesian approaches allow for flexible incorporation of prior knowledge and updating beliefs with new data. Discrete Bayesian models are increasingly applied in areas like natural language processing and bioinformatics.

2. High-Dimensional Discrete Data

Handling high-dimensional discrete data (e.g., categorical data with many levels) requires specialized modeling techniques, such as hierarchical models and graphical models.

3. Discrete Survival and Event Models

Extensions of survival analysis to discrete time frames facilitate modeling events like system failures or disease onset at specific intervals.

4. Machine Learning and Discrete Models

Algorithms such as decision trees, discrete latent variable models, and probabilistic graphical models leverage discrete probability methods for classification, clustering, and inference tasks.

Challenges and Future Perspectives

While discrete probability models are powerful, several challenges persist:

- Scalability: As data size and complexity grow, computational efficiency becomes critical.
- Model Selection: Choosing appropriate distributions and parameters requires robust criteria and methods.
- Interpretability: Ensuring models remain understandable for practical decision-making.
- Integration with Continuous Models: Hybrid models that combine discrete and continuous variables are increasingly relevant.

Future research is poised to focus on developing more efficient algorithms, richer models for complex data, and integrating discrete probability methods with other statistical and machine learning frameworks.

Conclusion

Discrete probability models and methods form a vital part of the statistical toolkit for analyzing countable random phenomena. Their theoretical elegance, combined with practical versatility, makes them indispensable across scientific disciplines. As data complexity continues to escalate, ongoing innovations in modeling techniques, computational algorithms, and applications will ensure their relevance and utility well into the future. Embracing these models' theoretical foundations and methodological advancements will remain essential for researchers and practitioners aiming to extract meaningful insights from discrete data.

QuestionAnswer What are discrete probability models and how are they used in probability theory? Discrete probability models are mathematical frameworks that describe the likelihood of different outcomes in scenarios where the possible outcomes are countable or finite. They are used to calculate probabilities, analyze distributions, and predict outcomes in applications like games, queues, or sampling processes. What is the significance of probability mass functions (PMFs) in

discrete models? Probability mass functions (PMFs) specify the probability of each possible outcome in a discrete random variable. They are fundamental in discrete probability models because they allow us to compute probabilities, expected values, and variances for discrete distributions. How do the Binomial and Poisson distributions serve as core discrete probability models? The Binomial distribution models the number of successes in a fixed number of independent Bernoulli trials with the same probability, while the Poisson distribution models the number of events occurring in a fixed interval of time or space when events happen independently at a constant average rate. Both are essential for modeling count data in various fields. What methods are commonly used to estimate parameters in discrete probability models? Methods such as Maximum Likelihood Estimation (MLE), Method of Moments, and Bayesian inference are commonly used to estimate parameters in discrete probability models, providing the best-fit values based on observed data. How can discrete probability models be applied in real-world scenarios? Discrete models are used in areas like quality control (defect counts), insurance (claim counts), queuing systems (number of customers), and genetics (number of genetic traits), helping to analyze, predict, and make decisions based on count-based data. What are the key assumptions underlying discrete probability models? Key assumptions typically include independence of events, fixed probabilities or rates, and the countable nature of outcomes. These assumptions ensure the models accurately reflect the stochastic processes they represent. How do discrete probability methods differ from continuous probability methods? Discrete probability methods deal with countable outcomes and use PMFs to assign probabilities, whereas continuous methods handle uncountably infinite outcomes using probability density functions (PDFs). The mathematical tools and interpretations differ accordingly. What are some challenges in working with discrete probability models and how can they be addressed? Challenges include small sample sizes, model misspecification, and dependencies between events. These can be addressed by collecting sufficient data, validating models through goodness-of-fit tests, and incorporating dependence structures or more complex models like Markov chains.

Related keywords: discrete probability, probability distributions, combinatorics, random variables, binomial distribution, geometric distribution, probability mass function, joint probability, Markov chains, probability theory

Read the full article online: [DISCRETE PROBABILITY MODELS AND METHODS PROBABILITY](#)